SUPERSONIC FLOW PAST AXISYMMETRIC BODY WITH STRONG LOCAL TWO-PHASE SURFACE INJECTION

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It is known [1-3] that in order to provide heat shield or to improve the aerodynamics of the body strong injection of cooling gas into the supersonic stream is utilized. Analysis of flow characteristics in the neighborhood of the solid body in the presence of strong single-phase injection and the effect of injection on the aerodynamic characteristics of some axisymmetric bodies are given, e.g., in [2-4]. Supersonic flow past a blunt-nosed axisymmetric body with blowing of a mixture of gas and solid particles through a porous segment in the leading edge region is considered in the present paper. Such a situation could occur in modeling the breakdown of the heat shield of a flight vehicle during its reentry into the thick layers of atmosphere and also in the case of forced introduction of particles in the flow of the injected gas in order to break up the leading edge shock and accordingly the variation in the drag of the body [5]. A description of the trajectory of the particles has been obtained as a result of numerical and analytical solution of the problem and their analysis is used to arrive at conclusions on their intersection and, consequently, also on the multiple-valued nature of the flow parameters in the neighborhood of the line dividing the external flow and the injected two-phase mixture. Sufficient conditions for multiple-valuedness have been analytically found which agree with numerical results. It has been established that with a change in composition of sufficiently small particles within the limits 0.1 to 0.6 by weight of the injected mixture the drag coefficient of the body does not change by more than 10%.

1. Formulation of the Problem. It is known [4] that with strong injection, when Reynolds numbers based on the parameters of the injected gas and free-stream fluid are much greater than one, the flow region between the shock wave and the body is represented in the form of two inviscid flow regions (shock layer and the layer of injected gases) separated by the mixing layer. In computing the aerodynamic characteristics of the flight vehicle the thin mixing layer, in which molecular momentum transfer is significant, is usually replaced by a contact surface discontinuity. Therefore the problem of supersonic flow past a body of revolution by a pure gas in the presence of localized two-phase injection reduces to the solution of the system of equations describing gasdynamics of two-phase monodispersion flow [6-9]:

$$\frac{\partial \mathbf{F}}{\partial t}+\frac{\partial \mathbf{P}}{\partial x}+\frac{\partial \mathbf{Q}}{\partial r}+\mathbf{G}=0, \quad p=(\gamma-1)\rho\varepsilon, \tag{1.1}$$

$$\mathbf{F}=\begin{bmatrix}\rho\\ \rho u\\ \rho v\\ \rho\varepsilon\\ \rho_s\\ \rho_s u_s\\ \rho_s v_s\\ \rho_s T_s\end{bmatrix},\quad \mathbf{P}=\begin{bmatrix}\rho u\\ \rho u^2+p\\ \rho uv\\ \rho u(\varepsilon+p/\rho)\\ \rho_s u_s\\ \rho_s u_s^2\\ \rho_s u_s v_s\\ \rho_s u_s T_s\end{bmatrix},\quad \mathbf{Q}=\begin{bmatrix}\rho v\\ \rho uv\\ \rho v^2+p\\ \rho v(\varepsilon+p/\rho)\\ \rho_s v_s\\ \rho_s u_s v_s\\ \rho_s v_s^2\\ \rho_s v_s T_s\end{bmatrix},\quad \mathbf{G}=\begin{bmatrix}\rho v r^{-1}\\ \rho uv r^{-1}+\rho_s f_x\\ \rho v^2 r^{-1}+\rho_s f_r\\ \rho v(\varepsilon+p/\rho)r^{-1}+\rho_s(c_s/c_p q+\mathbf{V}_s\cdot\mathbf{f})\\ \rho_s v_s r^{-1}\\ \rho_s u_s v_s r^{-1}-\rho_s f_x\\ \rho_s v_s^2 r^{-1}-\rho_s f_r\\ \rho_s v_s T_s r^{-1}-\rho_s q\end{bmatrix}.$$

Here u and v are components of gas velocity $\mathbf{V}$; u_s and v_s are the components of velocity vector of "fluid" particles $\mathbf{V}_s$; ρ and ρ_s are the fluid and particle densities, respectively; p

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is the pressure; $\varepsilon = e + (u^2 + v^2)/2$ is the sum of internal and kinetic energies of the gas; γ is the adiabatic index for the gas (frozen); T_s is the temperature of particles; x and r are the cylindrical coordinates; t is the time; indices x, r, and s correspond to projections of vector quantities on x and r axes and to parameters of "fluid" particles.

Expressions for drag f and heat flux q in the case of spherical particles have the form [10]

$$\mathbf{f} = \frac{c_D \tau_h}{c_{D0}\tau_v}(\mathbf{V} - \mathbf{V}_s), \quad q = \frac{\mathrm{Nu}\,\tau_h}{\mathrm{Nu}_0 \tau_T}(T - T_s), \tag{1.2}$$

where $\tau_v = \rho_s^0 d_s^2/18\mu$ is the dynamic relaxation time for particles: μ is the gas viscosity; d_s is the particle diameter; ρ_s^0 is the density of particles; $\tau_T = (3/2)\mathrm{Pr}(c_s/c_p)\tau_v$ is the thermal relaxation time for particles; c_s/c_p is the ratio of specific heats of the particles and gas; $\tau_T = R/U_{max,\infty}$ is the characteristic hydrodynamic time. Drag coefficient $c_{D0} = 24/\mathrm{Re}$ and Nusselt number $\mathrm{Nu}_0 = 2$ correspond to Stokes flow condition for particles, i.e., at Reynolds numbers $\mathrm{Re} = \rho|\mathbf{V} - \mathbf{V}_s|d_s/\mu_s \ll 1$. In order to determine the coefficients c_D and Nu which in general are functions of Mach number $M = |\mathbf{V} - \mathbf{V}_s|/a_0$ $(a_0 = \gamma p/\rho)$, Reynolds number Re, and Prandtl number $\mathrm{Pr} = c_p\mu/\lambda$ (λ is the thermal conductivity of the gas). The relations obtained on the basis of approximating experimental data, e.g., in [8-14], are used.

In the present paper the coefficients c_D and Nu were computed using following formulas for the case of two-phase injection in accordance with [13, 14]

$$c_D = c_{D0}\,[1 + 0.00026\mathrm{Re}^{1.38} + 0.197\mathrm{Re}^{0.63}], \qquad \mathrm{Nu} = \mathrm{Nu}_0 + 0.459\mathrm{Pr}^{0.33}\mathrm{Re}^{0.55}. \tag{1.3}$$

In view of the small transit time of particles in the shock wave region when compared to relaxation times τ_v and τ_T and the small volumetric composition of particles in the flow it is possible to neglect the effect of the dispersion phase on the gaseous phase behind the shock [8]. Hence parameters of the gaseous phase behind detached shock are found from Rankine–Hugoniot relations for perfect gas. It is required to satisfy pressure and normal velocity continuity at the surface of contact discontinuity. "Frozen" boundary conditions are specified at the porous surface of the body through which two-phase injection is effected [13]:

$$\begin{gathered}(\rho v_n)_w = g(s), \quad 0 \leqslant s \leqslant s_{\text{вд}}, \\ v_{nw} = v_w \cos\varphi, \quad \frac{\gamma_w}{\gamma_w - 1}\frac{p_w}{\rho_w} + \frac{v_w^2}{2} = H, \\ v_{snw} = v_{nw}, \quad T_{sw} = T_w, \quad \rho_{sw} = \frac{z_s}{1 - z_s}\rho_w.\end{gathered} \tag{1.4}$$

Here s_b is the length of the porous leading edge surface along the generatrix; v_w is magnitude of the injected gas velocity; φ is the angle between the normal to the injection surface and velocity vector $\mathbf{V}_w$; γ_w is the adiabatic index for the injected gas; $H = (1/2)h_{w0}/h_{\infty 0}$ (h_{w0}, $h_{\infty 0}$ are, respectively, the stagnation enthalpy of the injected gas and the free-stream); z_s is the composition of particles in the mixture by weight $[z_s = \rho_s/(\rho + \rho_s)]$. The usual no-slip condition is specified along the nonporous region near the leading edge. Since the characteristics of the system of equations (1.1) for the "fluid" particles represent trajectories of particles [12, 13], boundary conditions are specified only at the injection surface.

Flow parameters in the system of Eqs. (1.1) and boundary conditions are nondimensional quantities: The phase velocities are referred to the maximum free stream velocity $u_{max,\infty}$, phase density to the free-stream density ρ_∞, pressure to the dynamic pressure $\rho_\infty u_{max,\infty}^2$, phase temperatures to the temperature $T_{ch} = u_{max,\infty}^2/c_{p\infty}$, and linear dimensions to leading edge radius or the radius of the mid-section R.

2. Numerical and Analytical Investigation of the Problem in the Absence of Particle Effect on the Parameters of Gaseous Phase. Numerical results show that in the case of two-phase spherically symmetric flow [15] when the particle composition by weight $z_s \lesssim 0.2$, gas parameters vary by less than 10%. In their turn, studies carried out in [8] indicated that the use of different expressions for interaction coefficients leads to the conclusion that the variation in two-phase flow parameters is within 10%. Hence when $z_s \lesssim 0.2$ there is practically no sense in considering the influence of particles on the parameters of gas phase.

Thus, when particle composition by weight in the injected flow is small it is possible to solve the initial problem in two stages. The first stage is the solution of gasdynamic equations (1.1) excluding terms representing interphase exchange with corresponding boundary conditions. The determination of gas flow parameters in the supersonic flow past blunt-nosed body with injection was carried out using S. K. Godunov's method with explicit separation of bow shock and the surface of discontinuity [3]. In the second stage the stationary system of equations for the "fluid" particles is solved in the known steady flow field of the gaseous phase taking into account terms involving the interphase momentum and energy transfer (1.2) and is written in the characteristic form along streamlines:

$$\frac{dx}{dz}=\frac{u_s}{\sqrt{u_s^2+v_s^2}},\quad \frac{dr}{dz}=\frac{v_s}{\sqrt{u_s^2+v_s^2}},$$
$$u_s\frac{du_s}{dz}=\frac{f_x}{\sqrt{u_s^2+v_s^2}},\quad u_s\frac{dv_s}{dz}=\frac{f_r}{\sqrt{u_s^2+v_s^2}},\quad u_s\frac{dT_s}{dz}=\frac{q}{\sqrt{u_s^2+v_s^2}}, \tag{2.1}$$

where z is the arc length of the streamline. The system of equations (2.1) with initial conditions (1.4) was solved using fourth-order Runge–Kutta method with automatic selection of integration step for a given accuracy. The following are the computed results for the supersonic flow (M_∞ = 4.0, γ_∞ = 1.4) past a cone with a spherical nose and a cone with a nose described by the generatrix $x^{10} + r^{10} = 1$ in the presence of two-phase injection along the normal to the porous surface with parameters:

$$(\rho v_n)_w=0.5,\ H=0.5,\ \gamma_w=\gamma_\infty,\ \rho_s^0=3000,\ c_p/c_s=0.8,\ \Pr=0{,}7, \tag{2.2}$$

where S_b = 0.75 for the cone with spherical nose and S_b = 0.72 for the truncated cone.

Figure 1 shows the flow pattern near the spherical nose with two-phase injection of spherical particles of diameter $d_s = 10^{-3}$, where continuous lines represent the form of the bow shock, surface of contact discontinuity in the gas, and also the particle streamlines. Dashed line represents the sonic line constructed on the basis of gas parameters. The particle streamlines indicate that when the particle size is sufficiently large they pass through the injection layer and the shock wave and exit beyond the limits of integration of the initial system of equations. In this case the inertial force of particles is greater than viscous interaction between phases. It is worth noting that this mathematical model does not consider the effect of particles on the form of the shock wave (see [5]).

A different picture is observed with a reduction in the particle size. In this case (see Fig. 2a) when $d_s = 7.5\cdot 10^{-4}$ particles are slowed down in the shock layer, turn around and are carried downstream. Particles coming out in the neighborhood of the axis of symmetry also get into the injection layer where they are again stopped in the flow of the injected gas and are carried into the shock layer.

Figure 2b for this case shows variation in the magnitude of the difference in phase velocity vectors (curve 1) and also components u and u_s (curves 2 and 3, respectively) along the particle streamline with coordinates x = –1.0 and r = 0.045 where it leaves the surface of the body. The behavior of curves in Fig. 2b indicates appreciable velocity nonequilibrium of the two-phase flow along the streamline. The local minimum in curve 2 characterizing the change in velocity component u along the streamline shows that in this region streamlines of particles enter the injection layer where the velocity component u has a negative value.

Thus, it follows from an analysis of Fig. 2a, b that when $d_s = 7.5\cdot 10^{-4}$ streamlines of particles can intersect. The presence of nonuniqueness in velocities expressed in Eulerian variables requires refinement of the formulated problem. It is seen from an analysis of curves in Fig. 2a that the region of multiple values, i.e., the region where particle streamlines intersect, coincides with the order of magnitude of the dimensions of the disturbed flow region. As shown by computations, further reduction in particle diameter leads to a reduction in this region and at sufficiently small value it disappears. This is confirmed by the flow pattern (Fig. 3a and b) obtained with $d_s = 3\cdot 10^{-5}$ and the other parameters remaining as before. In this case an almost-equilibrium flow is observed, as seen from the behavior of curves in Fig. 3b, because of the increase in interactions between the phases. Here the curve 1 illustrates the behavior of the quantity $|\mathbf{V} - \mathbf{V}_s|$ along the streamline with the same exit coordinates and the curves 2 and 3 represent the change in velocity components u and u_s, respectively. Further reduction in particle size leads to an equilibrium flow when the particle and gas streamlines practically coincide.

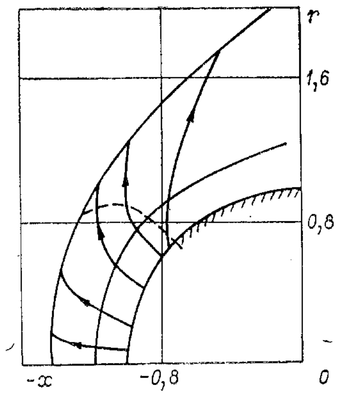

Fig. 1

Figure 4 shows the supersonic flow past a cone with a semivertex angle of 10°, the blunt nose being described by the generatrix $x^{10} + r^{10} = 1$, in the presence of an injection of a mixture of gas and particles with $d_s = 2.5 \cdot 10^{-4}$. It is seen that even in the case of supersonic flow past a truncated cone the quantitative flow pattern remains unaltered.

Analysis of this flow pattern for certain parameters indicates the presence of a dividing line between the flow region free of particles and the region of the shock layer in which particles are present. This line is the envelope of the family of particle streamlines penetrating through the injection layer into the shock layer. Consider the behavior of particles near this line in the coordinate system xOy associated with it. In this coordinate system equations of continuity and momentum for the "fluid" particles take the form

$$\frac{\partial}{\partial x}(\rho_s u_s r) + \frac{\partial}{\partial y}(\rho_s v_s r H_1) = 0, \quad H_1 = 1 + \varkappa y,$$
$$\frac{1}{H_1} u_s \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_s}{\partial y} + \frac{\varkappa u_s v_s}{H_1} = -\beta(u_s - u), \tag{2.3}$$
$$\frac{1}{H_1} u_s \frac{\partial v_s}{\partial x} + v_s \frac{\partial v_s}{\partial y} - \frac{\varkappa u_s^2}{H_1} = -\beta(v_s - v),$$

where $\varkappa$ is the curvature of the dividing line on which $v_s = 0$; $\beta = c_D \tau_h / (c_{D0} \tau_v)$.

At small values of $(-y)$, i.e., near the dividing line, the required function can be written in the form of a generalized power series

$$f(x, y) = f_0(x) + f_1(x)(-y)^k + \ldots$$

Then, since $v_s = 0$ at $y = 0$, the series for v_s will have the form

$$v_s(x, y) = v_{s1}(x)(-y)^n + \ldots$$

Substituting the major terms from the series in the third equation of the system (2.3) we get

$$u_{s0}(-y)^n \frac{\partial v_{s1}}{\partial x} - n v_{s1}^2 (-y)^{2n-1} - \varkappa u_{s0}^2 = -v_{s1}(-y)^n \beta + v_0 \beta. \tag{2.4}$$

Two cases are possible here depending on the value of the expression $\varkappa u_{s0}^2 + \beta v_0$.

Consider the case $\varkappa u_{s0}^2 + \beta v_0 < 0$. This inequality arises when the centrifugal force of particles is less than the resistance of the gas. Then from an analysis of Eq. (2.4) we get

$$n = \frac{1}{2}, \quad v_{s1}^2 = -2(\varkappa u_{s0}^2 + \beta v_0) > 0.$$

It is seen from this that when $\varkappa u_{s0}^2 + \beta v_0 < 0$, which occurs at least in the neighborhood of the axis of symmetry, the velocity component v_s of the particles at each point of the flow near the dividing line can have two values, equal in magnitude but opposite in direction. This physically means that in the given flow region there are particles not only moving in the direction of the shock wave but also in the opposite direction which is confirmed by the results of the above computations.

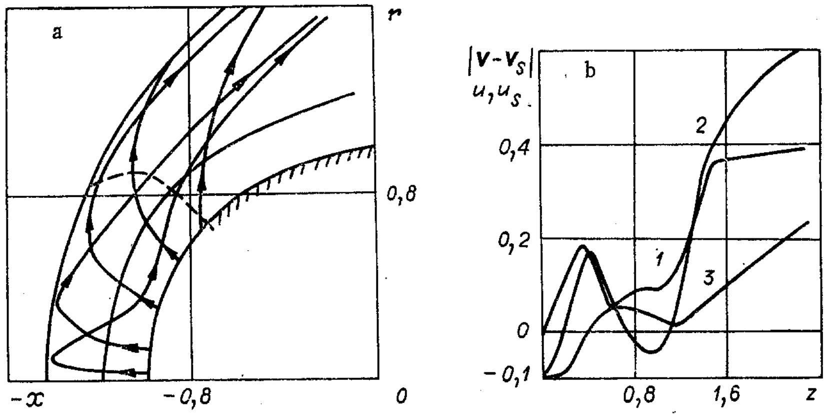

Fig. 2

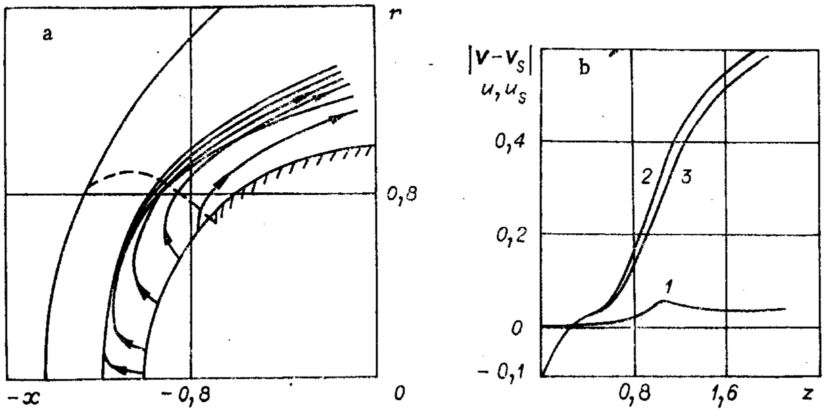

Fig. 3

Thus, in the region where the particle streamlines intersect there are at least three speeds, and one can refer to a third phase, viz., phase of particles "reflected" from the dividing line. In this case the basic system of equations (1.1) should be supplemented by equations for the particles of the third phase, and the equations for the gas by terms representing momentum and energy transfer with these particles.

Similar analysis of the equation of continuity for "fluid" particles gives the following expression for ρ_s as $(-y) \to 0$

$$\rho_s(x, y) = \rho_{s0}(x) + \rho_{s1}(x)\frac{1}{\sqrt{-y}} + \ldots$$

It follows that the particle density ρ_s tends to infinity when approaching the dividing line. Such a behavior of the quantity ρ_s is, apparently, the result of the initial assumption that particles do not interact with each other. This assumption is apparently not valid near the dividing line. The introduction of the third phase, viz., the particles "reflected" from the dividing line, requires the specification of boundary conditions for the system of equations describing the flow of this phase. Analysis of conservation laws at the dividing line gives

$$\text{as } y \to -0 \quad \rho_s v_s = -\rho_{s3} v_{s3}, \; v_{s3}/v_s = -1, \; v_s = 0,$$
$$T_{s3} = T_s, \qquad u_{s3} = u_s,$$

where index 3 indicates third phase. Thus, when $\varkappa u_{s0}^2 + \beta v_0 < 0$ the dividing line y = 0 is a weak discontinuity for the lifting phase and contact discontinuity for "fluid" particles.

If $\varkappa u_{s0}^2 + \beta v_0 = 0$, then in this case there is no ambiguity in the value of v_s. The particles reach the dividing line and then move along it. Unlike the previous case the derivatives with respect to y on either side of y = 0 are finite for all flow parameters.

3. Effect of Condensation Phase on the Aerodynamics of the Body. As shown by computations without the consideration of the reverse effect of particles on the parameters of the

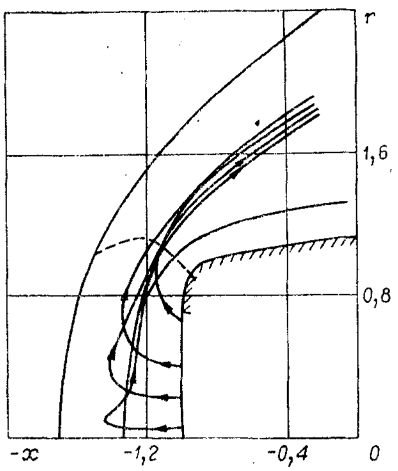

Fig. 4

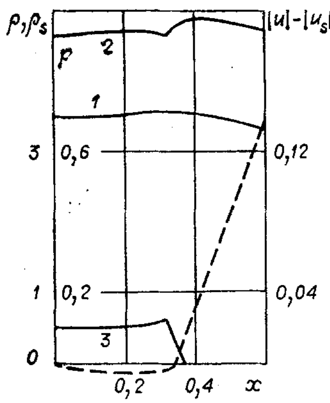

Fig. 5

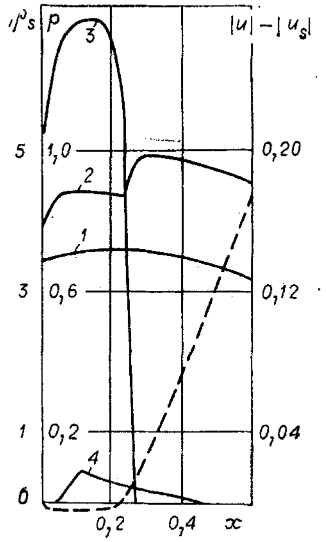

Fig. 6

lifting phase, when particle size is sufficiently small the interaction of particle streamlines practically disappears. In this case it is possible to use the system of equations (1.1)-(1.4) to describe mathematically two-phase injection in supersonic external flow. This problem was solved using S. K. Godunov's relaxation technique with the discontinuity surface in the gas phase. The procedure described in the method with donor cells [17] and the method of "flows" [18] was used to determine "large" values at the boundaries of neighboring cells for "gas" particles.

The computational results given below were obtained with following parameters: $M_\infty = 4.0$, $\gamma_\infty = \gamma_w = 1.4$, $[(\rho + \rho_s)v_n]_w = 0.5$, $s_b = 0.72$, $H = 0.5$, $Pr = 0.7$, $c_p/c_s = 0.8$, $\rho_s^0 = 3000$, $d_s = 3\cdot 10^{-5}$. Particle composition by weight z_s in the injected gas was varied from 0 to 0.7.

Figure 5 shows the variation of pressure p (curve 1), gas and particle densities (curves 2 and 3, respectively), and also the difference in the moduli of velocity components u and u_s (dashed line) along the axis of symmetry from the surface of the body to the shock wave with $z_s = 0.1$. The pressure in the injected layer ($x \leqslant 0.337$) is practically constant just as in the case of the absence of particles and falls linearly in the shock layer to its value on the shock wave. The density of particles ρ_s increases a little towards the surface dividing the external flow and the injected gas and then decreases sharply to zero. The curve for the difference in the width of gas and particle velocities $|u| - |u_s|$ shows that for sufficiently small particles the deceleration of the gas near the axis of symmetry is negligible and when $x > 0.337$, i.e., in the shock layer, this curve represents a change in the u-component of the gas.

With increase in the fraction of particles z_s by weight in the injected flow the geometric pattern of the flow does not quantitatively alter except for the sonic line. An increase in the parameter z_s makes it elongated downstream, i.e., in this case the gas velocity attains sonic velocity later due to the decelerating effect of the dispersion phase. An increase in particle weight composition also leads to a reduction in the density of the gas phase and an increase in the density of the mixture as a whole in the injected layer, which results in the reduction of injection speed and a consequent reduction in specific mass flow rate of the gas $(\rho v_n)_w$ for a constant two-phase injection. Therefore an increase in the parameter z_s leads to a reduction in the deviation of the contact surface in the gas by 28% and the shock wave by 11% with $z_s = 0.6$ when compared to the case $z_s = 0.1$.

In Fig. 6 for $z_s = 0.6$ curve 1 represents pressure distribution across the shock layer; 2 gives density distribution for the gas ρ along the axis of symmetry as a function of x representing the distance from the stagnation point to the shock wave; curve 3 and the dashed line are, respectively, the density distribution of the dispersion phase ρ_s and the difference in moduli of constituent velocities $|u| - |u_s|$ along the axis of symmetry; 4 is the particle density distribution along the last line of the computational scheme for the lateral surface of the cone to the shock wave. The behavior of this curve indicates that there is a layer of pure gas at the lateral surface of the cone and particles downstream are concentrated in a fairly narrow region when compared to the total thickness of the injected layer and the shock layer. It is seen from the behavior of curves 2 and 3 that in this case the density of the condensed phase became larger than the density of the gas phase and the density of the two-phase mixture more than doubled when compared to the case $z_s = 0.1$.

It is worth noting that the pressure distributions along the lateral surface of the body quantitatively differ very little in both cases. A certain increase in pressure is noticeable in the neighborhood of the point where injection ends with increase in which leads to an increase in wave-drag coefficient by 7.7% at $z_s = 0.6$ compared to the case $z_s = 0.1$ and in the total drag by 6% taking into account the thrust due to the injected flow. It is possible to conclude on the basis of computations that the fraction by weight of small-sized particles in the injected flow has insignificant effect on the drag of the body with change in z_s within the limits $0.1 \leqslant z_s \leqslant 0.6$.

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